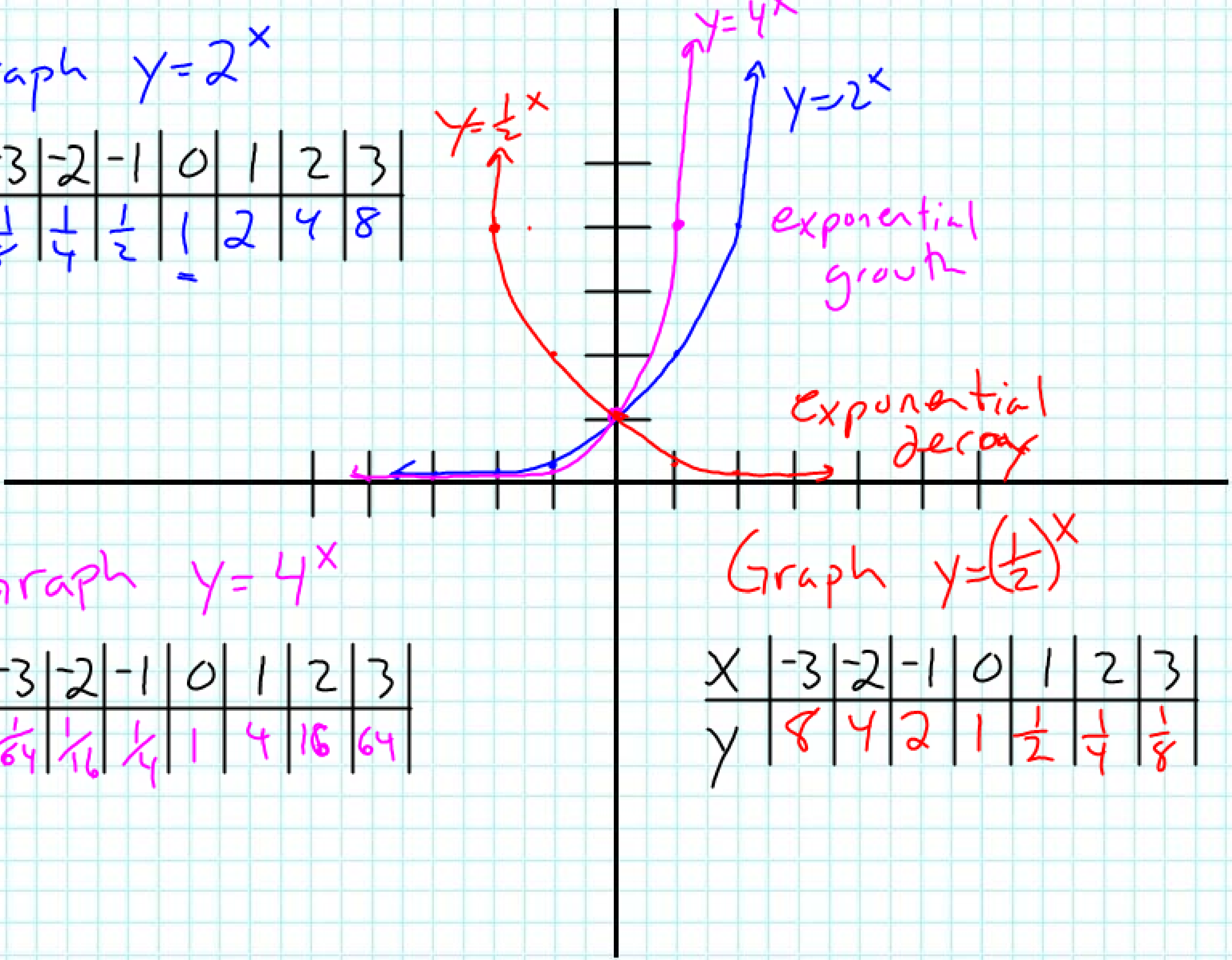
Exponential Function

$y = ab^x$, $a \neq 0$, $b > 0$, $b \neq 1$

Graph $y = 2^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Graph $y = 4^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64

Graph $y = \left(\frac{1}{2}\right)^x$

x	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

If $f(x) = 3^x$, sketch NO CALC.

(a) $g(x) = 3^{x+1}$ $f(x+1)$ horizontal shift
lft 1

(b) $g(x) = 3^x - 2$ $f(x) - 2$ vertical shift
down 2

(c) $g(x) = -3^x$ $-f(x)$ reflect over x-axis

(d) $g(x) = 3^{-x} \Rightarrow \frac{1}{3^x}$ $f(-x)$ reflect over y-axis

Growth factor $b = 1 + r$, r = rate of increase

$y = ab^x$

start value

often time

Exponential Decay $b < 1$

$b = (1 - \text{rate of decrease})$

6% decrease

$1000(1 - 0.06)^x$

$1000(0.94)^x$

$\cancel{1000(0.06)^x}$

Investments

- APR → Annual percentage rate %

- compounding

  monthly $\frac{APR}{12}$, your x is now months

Invest $5000 at 4% APR compounded monthly

$y = a\,b^x$ ← $1+r$

a ↓ start

$y = 5000\left(1 + \frac{0.04}{12}\right)^x$ x → months

or

$y = 5000\left(1 + \frac{0.04}{12}\right)^{12x}$ x = yrs

You invest \$1.00 at 100% APR for 1 year. What is your balance at the end of the year if you compound the interest Keep lots of decimal places

a) Yearly? 2.00

b) Quarterly? 2.4414

c) Monthly? 2.61303

d) Weekly? 2.6926

e) Daily? 2.7146

f) Every hour? 2.7181

g) Every minute? 2.718279 $\longrightarrow \approx e$

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

e - natural base

$$A = P(1+r)^t$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$m = \frac{n}{r}$ $\quad \frac{1}{m} = \frac{r}{n}$

$n = mr$

$$A = P\left(1 + \frac{1}{m}\right)^{mrt}$$

$$A = P\left(\underline{\left(1+\frac{1}{m}\right)^m}\right)^{rt}$$

$$\boxed{A = Pe^{rt}}$$

A → balance

P → initial investment

r → rate (like 0.05)

t → time in yrs.

Continuous Compounding

To compound continuously, we use the natural base (e). We most often use this with natural events of growth (e.g. population) or decay.

$$y = a e^{bx}$$

or as we did in class for an investment

$$A = Pe^{rt}$$

- A: ending amount
- P: start value (principal)
- e: base e
- r: the rate
- t: usually years

Ex. \$3,000 invested at 4.5% APR compounded continuously.

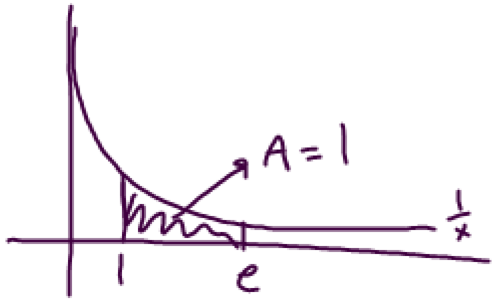
$$A = 3000e^{(0.045 \cdot t)}$$

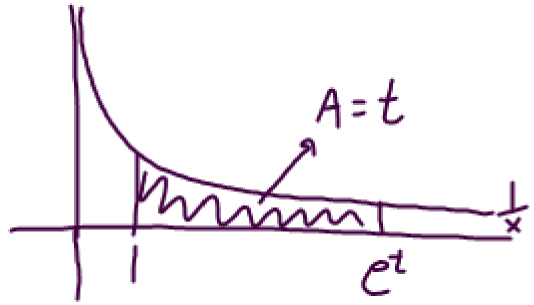
For 10 years later: $A = 3000e^{(0.045 \cdot 10)}$

$$\boxed{\approx \$4{,}704.94}$$

More on e and e^x

- Area bounded by $f(x)=\frac{1}{x}$ and x-axis and 1 and e $=1$
  and 1 and $e^t = t$





- derivitive and integral of $e^x = e^x$

- you can change any equation $y=ab^x$ to base e by writing b in the form $b=e^{\ln b}$

For example, $y=3(2)^x \Rightarrow y=3e^{(\ln 2)x} \approx 3e^{0.693x}$

Half-Life

You have \$75,000 in a retirement account.
Your account loses half its value every 5 years.

ⓐ Write a model for this situation.

$y = ab^x$

$75000(1-0.5)^{\frac{x}{5}}$ — every 5 yrs

ⓑ Find the value after 9 years.

21,538

The half-life of a radioactive substance is the time it takes for half of the material to decay. A hospital prepares a 100mg supply of technetium-99m, which has a half-life of 6 hours. Write an exponential function for the amount of technetium-99m after x hours and then find the amount remaining after 75 hours.

$$f(x) = 100\left(\frac{1}{2}\right)^{\frac{x}{6}} \quad x \rightarrow \text{hours}$$

$$f(75) = 100\left(\frac{1}{2}\right)^{\frac{75}{6}} \approx 0.02\text{mg}$$

Writing an exponential equation through two points

$y = ab^x$ (2, 2) (3, 4)

① $2 = ab^2$

② $\frac{2}{b^2} = \frac{ab^2}{b^2}$

$\frac{2}{b^2} = a$

③ $y = ab^x$

$4 = \frac{2}{b^2} \cdot b^3$

$4 = \frac{2b^3}{b^2}$

$\frac{4}{2} = \frac{2b}{2}$

$2 = b$

④ $\frac{2}{b^2} = a \rightarrow \frac{2}{2^2} = a$

$a = \frac{1}{2}$

Steps

① Start with $y = ab^x$, plug in first pt. for x + y

② Solve for a

③ Plug in a, and 2nd pt.

$y = ab^x$

solve for b

④ Sub. your b back into the a equation to find a

⑤ Write equation using a + b

$$\boxed{y = \frac{1}{2}(2)^x}$$

Write an equation, $y = ab^x$, through $(2,4)(3,16)$

Logarithmic Function

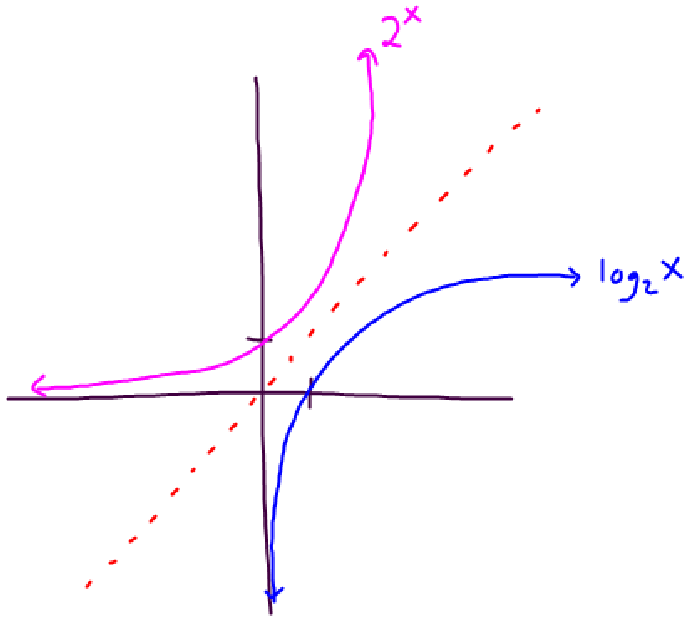
$y = \log_a x$ if and only if $x = a^y$

$x > 0,\ a > 0,\ a \neq 1$

$f(x) = \log_a x$ → value (Ans to exponential equation)

$f(x)$ ↓ exponent

a ↓ base



* The log is the inverse of the exponential function

* If it just has $y = \log x$ with no base listed, the base is 10

* $y = \log_e x \Rightarrow y = \ln x$

Properties of Logs - Basic

1. $\log_a 1 = 0$ b/c $a^0 = 1$

2. $\log_a a = 1$ b/c $a^1 = a$

3. $\log_a a^x = x$ b/c $a^{\log_a x} = x$

4. If $\log_a x = \log_a y$ then $x = y$

All these apply to natural log, ln, as well

More Properties

① $\log_a(uv) = \log_a(u) + \log_a(v)$

② $\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$

③ $\log_a(u^n) = n\log_a(u)$

All the same properties hold with the natural log, ln

Examples

① Expand

$\log_4 5x^3 y$

$\downarrow$

$\log_4 5 + \log_4 x^3 + \log_4 y$

$= \log_4 5 + 3\log_4 x + \log_4 y$

② Condense

$2\ln(x+2) - \ln x$

$\ln(x+2)^2 - \ln x$

$= \ln \frac{(x+2)^2}{x}$

You Try

① Expand $\ln \frac{\sqrt{3x-5}}{7}$

② Condense $\frac{1}{3}\left[\log_2 x + \log_2 (x-4)\right]$

All you really need

$$7^x = 12$$

$$\log_7(12) = x$$

$$x = \frac{\log 12}{\log 7}$$

To graph $\log_5 x$ on calc $\rightarrow$ $y = \dfrac{\log x}{\log 5}$